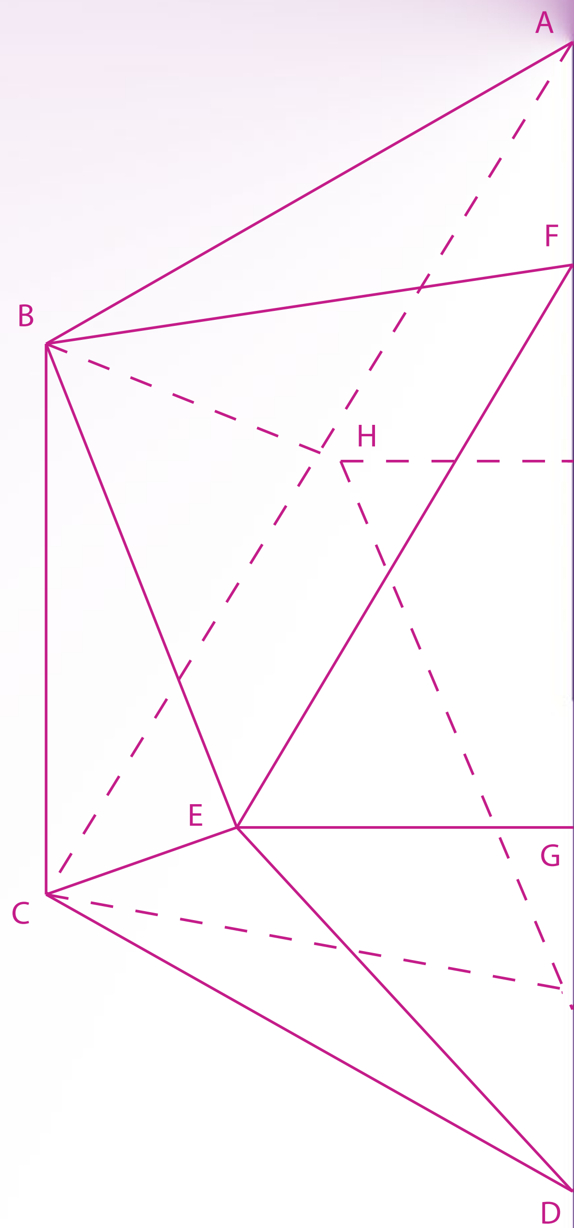

Magoosh

SAT Math Formula eBook

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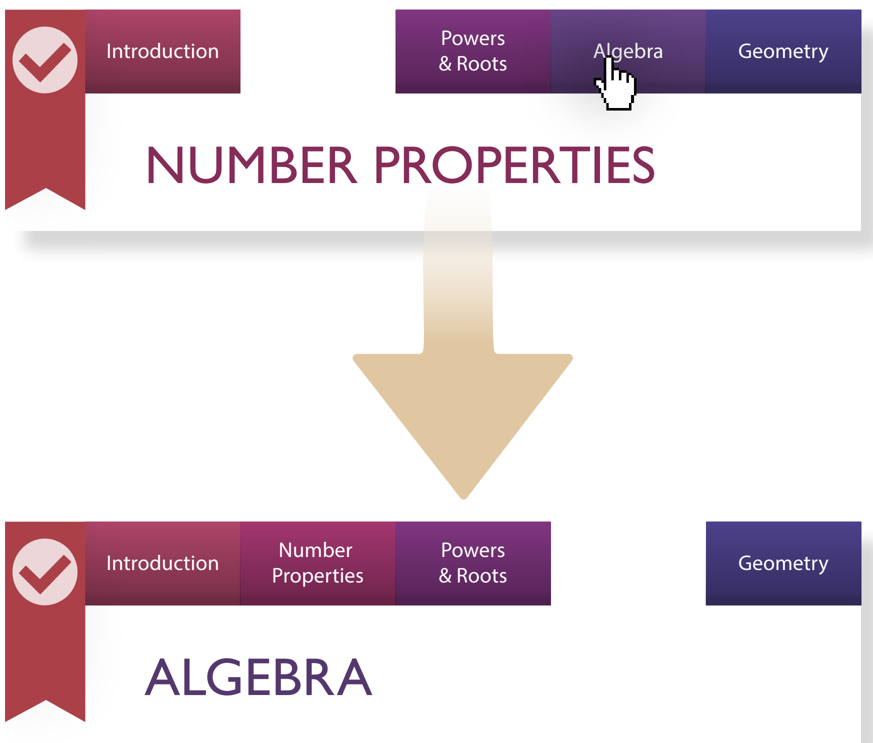
How to Use this eBook

This ebook is a study tool that combines all the math formulas that you'll need to know for the SAT in one handy place. If a math concept is tested on the SAT, you'll find any relevant formulas here, along with helpful tips for using the formula quickly and accurately.

In addition to formulas, we've also included some excerpts from the Magoosh SAT blog. They provide relevant study strategies and time-saving tips. And don't forget to use our practice problems to quiz yourself along the way!

You'll also notice there are tabs along the top of the page. You can use these tabs to move between the book's main sections. For example, if you need to brush up on your number properties, just click the number properties tab at the top of the page and—voilà—you are in the number properties section! Done with number properties, and want to try some algebra? Just click the algebra tab and you're there! No scrolling required.



Basically, we want you to think of this eBook not only as a formula cheat-sheet, but also as a great reference for all SAT math topics that you can use at any stage of your study process. We hope that it's helpful and informative! If you have any questions, comments, or suggestions, please feel free to leave us a comment on our **blog**!

INTRODUCTION

About Us

Magoosh is an online SAT prep course that offers:

- 140+ unique lessons on all SAT subjects
- 700+ Math & Verbal practice questions, with video explanations after every question
- Material created by expert tutors, who have in-depth knowledge of the SAT
- Free vocabulary flashcards online and through our Android and iPhone Mobile Apps!
- Access anytime, anywhere from an internet-connected device
- 150-point score-improvement guarantee
- Email support from experienced SAT tutors
- Customizable practice sessions and quizzes
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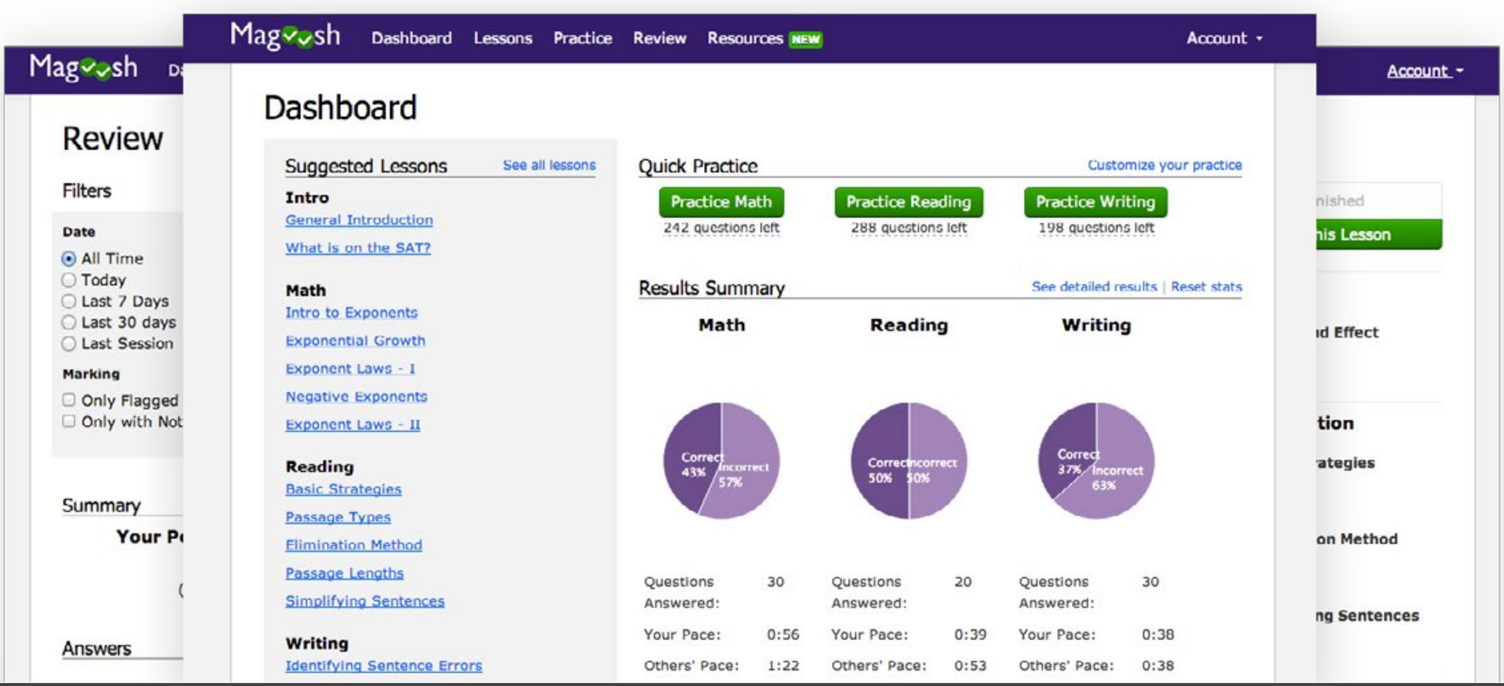


INTRODUCTION

What Students Say About Magoosh

“It has given me great preparation for hard questions on the SAT. It’s an amazing way to practice hard questions that you rarely find, and the answer explanations are amazing.”

“Unlike others out there, Magoosh has all the helpful, easy-to-comprehend video lessons. It helps a lot just by watching them.”

“I loved the videos and all the practice questions. The practice questions are all at (what I feel is) a harder level which really helped with level 4 and 5 type questions on the SAT. I have improved my English grammar skills significantly and excelled through lots of the practice questions. I also really love the videos at the end of each question. (: “

“Magoosh has not only helped me refresh for my upcoming test but also taught me some cool new tricks for quickly solving problems in all subjects. I would recommend Magoosh to anyone who wants to feel confident on the day of their test. “

“I used the product to prepare me for taking the SAT and even though I had taken it before, I was still very much informed on many things I did not know about the test. “

The Magoosh Team

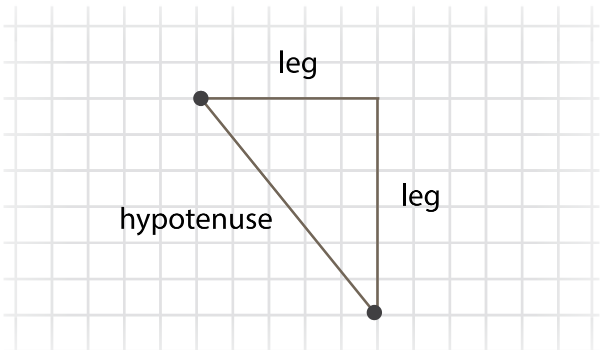
INTRODUCTION

SAT Math Formulas: How to (Not) Use Them

While formulas can be really helpful on the SAT, there are very, *very* few that you absolutely need to have memorized to score well. That might come as a surprise, but it's true, and it leads us to an important thought: understanding how and why a formula works is as useful as rote memorization. In fact, it's much better. You'll have a better sense of when to use a formula and be more accurate in executing it if you understand the math behind it. Let's look at a concrete case to illustrate. The distance formula is a prime example. It's ugly…

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

…but it actually represents a pretty simple idea. If you have any two points on a graph (on the coordinate plane), you can make a right triangle that connects those two points as the ends of the hypotenuse. That is, you draw a diagonal line between the two points, then a straight horizontal line and a straight vertical line going through each point to make the legs of the triangle.



Then, since you're trying to find the length of the hypotenuse, you just use the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

(Notice that a couple very basic formulas like this one do need to be memorized.) The lengths of those legs are a and b, and the length of the hypotenuse is c.

So let's find the length of c:

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$

And if you're trying to find the length of the legs (the shorter sides), you just need to know the horizontal distance between the two points, $x_2 - x_1$, and the vertical distance between the two points, $y_2 - y_1$. If you replace a and b with those values, voilà: you have the distance formula.

INTRODUCTION

Additionally, relying on formulas too much can also give us formula blindness. That is, even though we've remembered a formula, we try to apply it to a problem even when the problem is asking for something different. The reason students often fall into this trap is because a question may use language that is similar to the language they'd expect for using a certain formula.

Once you've memorized the formulas in this eBook, you should practice them on relevant problems so that applying the formulas becomes natural. You should also be aware of when the formula doesn't completely apply. Or when you find a way to solve the problem without the formula, that's even better—just in case you happen to forget a formula on test day.

Takeaway

Ultimately, the SAT is testing the way you think. And simply plugging in a bunch of values to a set formula doesn't test thinking skills insomuch as it tests your ability to memorize a formula. Besides, trying to memorize a formula is often more difficult than knowing how that formula was derived. Nevertheless, before walking into the SAT, it is a good idea to know the following formulas and concepts. In fact, ignoring the information below can seriously hurt your chances of answering a question correctly. Just be sure to apply the formulas often so they're easy to remember and understand!

NUMBER PROPERTIES

Types of Numbers

- **Integers:** Any counting number, including negative numbers (*e.g.* -3, 0, 2, 7… but not 2.5)
- **Real Numbers:** Numbers that appear on the number line including π, $\sqrt{2}$, etc. (*i.e.*, ones that are not imaginary)
- A positive number is greater than 0; a negative number is less than 0.
- 0 is neither positive nor negative.

Order of Operations: PEMDAS

Complete any arithmetic operations in the following order:

1. Parentheses
2. Exponents
3. Multiplication / Division
4. Addition / Subtraction

Mutiplication/Division and Addition/Subtract are left to right.

Example:

$$2 + \frac{6}{2} \times (5 - 1)^2$$

$$2 + \frac{6}{2} \times (4)^2$$

$$2 + \frac{6}{2} \times 16$$

$$2 + 48$$

$$50$$

You can remember PEMDAS as "Please Excuse My Dear Aunt Sally," or "Purple Eggplants Make Delicious Afternoon Snacks," or my personal favorite, "Pandas Explore Many Delightful Asian Scenes"

Commutative, Associative, & Distributive Properties

- The Commutative Property: $a + b = b + a$
  $a \times b = b \times a$
- The Associative Property: $(a + b) + c = a + (b + c)$
  $(a \times b) \times c = a \times (b \times c)$
- The Distributive Property: $a \times (b + c) = ab + ac$
  $a \times (b - c) = ab - ac$

The Commutative and Associative properties do not work with subtraction or division.

NUMBER PROPERTIES

Prime Numbers

A prime number is one that is divisible only by itself and 1. In other words, a positive integer with exactly two positive divisors. This includes 2, 3, 5, 7, and 11, but not 9, because $9 = 3 \times 3$.

- 1 is not a prime.
- 2 is the smallest prime and the only even prime.
- 0 and negatives aren't prime.
- Memorize all primes below 20: 2, 3, 5, 7, 11, 13, 17, 19

Factorization

If x can be multiplied by y to get z, assuming all of these are positive integers, then x and y are considered **factors** of z.

- You get the prime factorization of a number by splitting it into the primes that multiply into it. So for 21, this is 3×7; for 60, it's $2 \times 2 \times 3 \times 5$.
- To find the prime factorization you can break the number up one step at a time. For example, $60 = 30 \times 2 = 15 \times 2 \times 2 = 5 \times 3 \times 2 \times 2$.
- To find how many factors 720 has, first find its prime factorization: $2^4 \times 3^2 \times 5$. All of its factors will be of the form $2^a \times 3^b \times 5^c$. Now there are five choices for a (a= 0, 1, 2, 3, or 4), three choices for b (b = 0, 1, or 2), and two choices for c (c = 0 or 1). The total number of factors is therefore $5 \times 3 \times 2 = 30$. 720 has 30 factors. To generalize, the number of divisors of any number is $(a+1) \times (b+1) \times (c+1)$... where a, b, and c are the exponents above the prime factors of the number.

Greatest Common Factor

The greatest common factor (aka greatest common divisor) of two numbers is the biggest factor they share. For example, the GCF of 12 and 30 is 6—it is the biggest divisor they both share.

- The easiest way to find the GCF is to take the prime factoriztion and multiply all of the primes that appear in both numbers.

  So since **56** = 2 × 2 × **2** × **7** and 70 = **2** × 5 × **7**, the GCF is $2 \times 7 = 14$. If two numbers share no primes, the GCF is 1.

Least Common Multiple

The least common multiple of two numbers is the smallest positive integer with both numbers as a factor. The LCM of 4 and 6 is 12—it is the smallest number that has both 4 and 6 in its divisors. Similarly, the LCM of 9 and 15 is 45; the LCM of 7 and 21 is 21, because 21's factors are 1, 3, 7, and 21.

NUMBER PROPERTIES

- To find the LCM of any two numbers, take the prime factorization of each number, find what prime factors appear in both lists, then multiply the shared prime factors by those primes that aren't shared in each list.

  So for example, $12 = 2 \times 2 \times 3$, and $56 = 2 \times 2 \times 2 \times 7$, so we first notice that both lists share 2×2. Then, we multiply that by the numbers that aren't shared from each: 3 from 12's prime factors and (2×7) from 56's primes factors. That's (2×2) [shared] $\times$ (3) [unshared] $\times$ (2×7) [unshared] $= 2 \times 2 \times 3 \times 2 \times 7 = 168$.
- The largest possible LCM for any two numbers is one number multiplied by the other.

Divisibility

How to tell if a number is divisible by…

- 3 : sum of digits divisible by 3
- 4 : the last two digits of number are divisible by 4
- 5 : the last digit is either a 5 or zero
- 6 : even number and sum of digits is divisible by 3
- 8 : if the last three digits are divisible by 8
- 9 : sum of digits is divisible by 9

Fast Fractions

$$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} \longrightarrow \frac{1}{2} + \frac{1}{5} = \frac{2+5}{2 \times 5} = \frac{7}{10}$$

Absolute Values

The absolute value of a number is its distance from zero on the number line.
For any positive x…
$|x| = x$, $|-x| = x$
And for any negative x…
$|x| = -x$, $|-x| = -x$
…since $-x$ would represent a positive number (the negative of a negative). Remember that x is negative to begin with it, so in order to make x positive we actually have to have $-x$.

NUMBER PROPERTIES

Percentages

"Percent" = per 100

$19\% = \frac{19}{100}$

$.43\% = \frac{.43}{100} = \frac{43}{10000}$

- To find what percent some part is of a whole, use $\frac{part}{whole} = \frac{percent}{100}$.

  For example, if 120 of 800 people in a town smoke, then $\frac{120}{800} = \frac{percent}{100} = \frac{15}{100} \longrightarrow 15\%$ of the townspeople smoke. Most percentage problems break down into identifying either the part, the percent, or the whole. One of these will be unknown and the other two will be known.

- Percent change : $\% \text{ change} = \frac{\text{change}}{\text{original value}}$
  - If the price of something goes from $40 to $52, the percent change is $\frac{(52 - 40)}{40} = \frac{12}{40} = \frac{3}{10} = \frac{30}{100} = 30\%$. The price increases by 30%.
  - This can also be written as (change x 100) / original value. So here, it's $\frac{(52 - 40) \times 100}{40} = \frac{1200}{40} = 30\%$.
  - If something increases by 20%, then decreases by 5%, it is not the same as if it increased by 15%. For example: 100 -> 120 -> 114, whereas if 100 increased by 15% it would be 115.
  - Let's say we know the percent that something went down and want to know the new value. We change the percent into a decimal and subtract it from 1. For example, if a price dropped by 15%, we would do the following: 1 - .15 = 0.85. Now we take the new price and multiply it by 0.85.
  - 250% of the original price is the same as 150% more than the original price, and to find either you'd multiply the original price by 2.5.

Ratios

Ratios let us compare the proportions of two quantities. If there is a 2:5 ratio of boys to girls at a school, that means that for every 5 girls, there are 2 boys. So there could be 2 boys and 5 girls, 20 boys and 50 girls, 200 boys and 500 girls, *etc*.

- Ratios are given as x:y, x to y, or x/y. If a question says "for every x there is/are y," you are most likely dealing with a ratio question.
- Ratios can be simplified like fractions. 3:6 is the same as 1:2.
  - Ratios can also be in three parts, as in x:y:z.

NUMBER PROPERTIES

- To combine ratios, create a common term. For example, $2a$:$3b$ and $2b$:$3c$ can be combined by converting the b term to $6b$, which works well in either ratio— $4a$:$6b$ and $6b$:$9c$ preserve both relationships, but now they can be combined into $4a$:$6b$:$9c$. This also tells us that the ratio of a to c is $4a$:$9c$.
- Remember that if there is a 2:5 ratio of boys to girls at a school, the ratio of boys to total students is 2:(5 + 2) = 2:7. 2/7 of the students are boys.

Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

1. If k is an integer, what is the smallest possible value of k such that $1040k$ is the square of an integer?
   a. 2
   b. 5
   c. 10
   d. 15
   e. 65

2. A retailer purchases shirts from a wholesaler and then sells the shirts in her store at a retail price that is 80 percent greater than the wholesale price. If the retailer decreases the retail price by 30 percent, this will have the same effect as increasing the wholesale price by what percent?
   a. 26
   b. 37.5
   c. 42
   d. 44
   e. 50

3. In a certain town in Connecticut, the ratio of NY Yankees fans to NY Mets fans is 3:2, and the ratio of NY Mets fans to Boston Red Sox fans is 4:5. If there are 300 baseball fans in the town, each of whom is a fan of exactly one of those three teams, how many NY Mets fans are there in this town?
   a. 75
   b. 80
   c. 90
   d. 120
   e. 133

NUMBER PROPERTIES

Answers and Explanations

Click the play button for a video explanation.

1. Answer: e. 65

$$1040 = 2 \times 2 \times 2 \times 2 \times 5 \times 13$$
$$1040k = 2 \times 2 \times 2 \times 2 \times 5 \times 13 \times k$$
$$= (2 \times 2 \times 5 \times 13)(2 \times 2 \times k)$$
$$k = 5 \times 13$$
$$k = 65$$

2. Answer: a. 26

Wholesale: \$100 → +80% (+\$80) → Retail: \$180 → -30% (-\$54) → Reduced: \$126

\$100 ⟶ \$126 = 26% increase

3. Answer: b. 80

The ratio of iniquitous Yankees fans to pure-hearted Mets fans is 3:2, and the ratio of these same noble Mets fans to the misguided Red Sox fans is 4:5. (*The* author *acknowledges no preferences in this description*.) We need to combine these into one ratio, but in order to do that, we need to make the common term, Mets fans, equal in both.

Fortunately, Mets fans are represented by 2 in the first ratio and 4 in the second ratio, so we just have to multiply the first ratio by two—a 3:2 ratio is the same thing as a 6:4 ratio. Now we can combine these into a single 6:4:5 ratio—6 parts Yankees fans, 4 parts Mets fans, and 5 parts Red Sox fans. That's a total of 15 parts in the whole population. Well, there are 300 baseball fans, and if that's 15 parts, each part is worth 300/15 = 20 fans. The Mets fans comprise 4 parts, or 4 × 20 = 80 fans, answer = **b**. Fewer than the fans of the other two, less virtuous teams: many are called, few are chosen.

POWERS & ROOTS

Exponents

Notation:

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
$x^3 = x \times x \times x$
$4^x = 4 \times 4 \times 4 \ldots (x \text{ multiples of } 4)$
$x^1 = x$

Exponent Laws:

- $x^A \times x^B = x^{(A+B)}$
- $\frac{x^A}{x^B} = x^{(A-B)}$
- $(x^A)^B = x^{(A+B)}$

1 and 0 as bases:

- 1 raised to any power is 1.
- 0 raised to any non-zero power is 0.
- Any non-zero number to the power of 0 is 1:

  $7^0 = 1$

  $54^0 = 1$

  $5 \times (x^0) = 5$

Fractions as exponents:

- $x^{\left(\frac{1}{2}\right)} = \sqrt{x}$
- $x^{\left(\frac{2}{3}\right)} = \sqrt[3]{x^2}$

Negative exponents:

- $x^{(-1)} = \frac{1}{x}$
- $x^{(-2)} = \frac{1}{(x^2)}$

Negative bases:

- A negative number raised to an even power is positive.
- A negative number raised to an odd power is negative.

  $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

  $(-2)^5 = -32$

Odd/even exponents:

$x^3 = 8 \longrightarrow x = 2$, but $x^4 = 16 \longrightarrow x = 2$ *and* $x = -2$

To raise 10 to any power, just put that many 0s after the 1: $10^5 = 100000$, a 1 with 5 zeros.

POWERS & ROOTS

Roots

$\sqrt{49} = 7$, because $7^2 = 49$. Note that even though $(-7)^2 = 49$ as well, -7 is NOT considered a solution of $\sqrt{49}$; only the positive number counts in this case. However, if you were given the question $x^2 = 49$, the answer would be $x = 7$ or $x = -7$.

Perfect squares:

Numbers with integers as their square roots: 4, 9, 16, etc.

- To estimate square roots of numbers that aren't perfect squares, just examine the nearby perfect squares. For example, to find $\sqrt{50}$, you know that $\sqrt{49} = 7$ and $\sqrt{64} = 8$, so $\sqrt{50}$ must be between 7 and 8.

Cube roots:

- $\sqrt[3]{n}$ = a number that, when cubed, equals n. $\sqrt[3]{-8} = -2$.

Simplifying roots:

- Separate the number into its prime factors, and take out matching pairs:

$$\sqrt{20} = \sqrt{2 \times 2 \times 5} = 2\sqrt{5}$$

Adding and Subtracting Roots:

- $2\sqrt{7} + 9\sqrt{7} = 11\sqrt{7}$. Roots can be added like variables.

Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

1. $\frac{4^6 - 4^5}{3} =$
   a. $\frac{4}{3}$
   b. $4^{4/3}$
   c. $4^4 - 4^{5/3}$
   d. $4^5 - 4^4$
   e. 4^5

2. If $k \neq 0$, $k \neq \pm 1$, and $\frac{(k^3 \times k \times k^4)^2}{k \times k} = k^{14}$, then $n =$
   a. -1
   b. 1
   c. 3
   d. 49
   e. 129

POWERS & ROOTS

Answers and Explanations

Click the play button for a video explanation.

1. Answer: e. 4^5

$$\frac{4^6 - 4^5}{3} = \frac{4^5(4^1 - 1)}{3}$$

$$= \frac{4^5(3)}{3}$$

$$= 4^5$$

2. Answer: b. 1

$$\frac{(k^3 \times k^1 \times k^4)^2}{k^n \times k^1} = k^{14} \qquad (x^a)(x^b) = x^{a+b}$$

$$\frac{(k^8)^2}{k^{n+1}} = k^{14} \qquad (x^a)^b = x^{ab}$$

$$\frac{k^{16}}{k^{n+1}} = k^{14} \qquad \frac{x^a}{x^b} = x^{a-b}$$

$$k^{16-(n+1)} = k^{14} \qquad x^a = x^b \longrightarrow a = b$$

$$16 - (n+1) = 14$$

$$15 - n = 14$$

$$n = 1$$

ALGEBRA

Simplifying Expressions

Simplifying expressions:

- $(6xy + 5x) - (4xy - 3y) = 2xy + 5x + 3y$

Multiplying monomials:

- $(5y^3)(6y^2) = 30y^5$
- $-6y(5x + 3y) = -30xy - 18y^2$

FOIL (First, Outer, Inner, Last):

- $(x + 2)(x + 7) = (x \times x) + (x \times 7) + (2 \times x) + (2 \times 7) = x^2 + 9x + 14$

Common patterns to memorize:

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a^2 - b^2) = (a + b)(a - b)$
- $(a - b)^2 = a^2 - 2ab - b^2$

Factoring

Factoring using Greatest Common Factors:

- $6x^3 + 12x^2 + 33x = 3x(2x^2 + 4x + 11)$

Factoring using the difference of squares:

- $(a + b)(a - b) = a^2 - b^2$
- $(2x + 5)(2x - 5) = 4x^2 - 25$
- $(2x - 3y)(2x + 3y) = 4x^2 - 9y^2$

Factoring using quadratic polynomials:

- $x^2 + ax + b = (x + m)(x + n)$, where a is the sum of m and n, and b is their product. For example, $x^2 + 5x - 14 = (x + 7)(x - 2)$

Examples using multiple methods:

- $2x^6 - 2x^2 = 2x^2(x^4 - 1) = 2x^2(x^2 + 1)(x^2 - 1) = 2x^2(x^2 + 1)(x + 1)(x - 1)$
- $9x^3y^2 - 6x^2y^2 + xy^2 = xy^2(9x^2 - 6x + 1) = xy^2(3x - 1)^2$

Factoring rational expressions:

- $\frac{6x^2 + 12x - 144}{2x^2 - 32} = \frac{6(x^2 + 2x - 24)}{2(x^2 - 16)} = \frac{3(x + 6)(x - 4)}{(x + 4)(x - 4)} = \frac{3(x + 6)}{x + 4}$
- The above example is only possible if $x \neq 4$ and $x \neq -4$

ALGEBRA

Solving Equations

The golden rule of solving equations is that what you do to one side of an equation, you must also do to the other.

Eliminating fractions:

- $\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) = 1$
- $\frac{2}{5}x = 8 \longrightarrow \frac{5}{2} \times \frac{2}{5}x = \frac{5}{2} \times 8 \longrightarrow x = 20$

Multiply by the LCD:

- $\frac{3x}{4} + \frac{1}{2} = \frac{x}{3} \longrightarrow \times 12 \longrightarrow \frac{36x}{4} + \frac{12}{2} = \frac{12x}{3} \longrightarrow 9x + 6 = 4x \longrightarrow x = -\frac{6}{5}$

Cross-multiplication:

- $\frac{a}{b} = \frac{c}{d} \longrightarrow ad = bc$
- $\frac{7}{6x - 6} = \frac{3}{2x + 2} \longrightarrow 3(6x - 6) = 7(2x + 2)$

Quadratic equations:

- For $ax^2 + bx + c$, where a is not 0, if you can factor it to $(x + y)(x - z)$, then the solutions are $-y$ and z. For example:

  $x^2 - 7x = -10$

  $x^2 - 7x + 10 = 0$

  $(x - 2)(x - 5) = 0$

  $(x - 2) = 0$

  $x = -2$

  or

  $(x - 5) = 0$

  $x = 5$

  $x = 2$ or $x = 5$

Two variables/systems of equations:

For example: $3x + y = 17$ and $2x - 2y = 6$

- Method 1: Substitution

  $y = 17 - 3x$

  $2x - 2(17 - 3x) = 6$

  $2x + 6x - 34 = 6$

  $x = 5$

ALGEBRA

- Method 2: Elimination

  $6x + 2y = 34$

  $2x - 2y = 6$

  add the two equations, so +2y and -2y eliminate each other

  $8x = 40$

  $x = 5$
- A system of two equations with two unknowns can have 0, 1, or infinite solutions.
- To solve a system of three equations with three variables, use substitution to reduce the problem to two equations with two variables, and solve from there.

Function notation

- If given $f(x) =$ ... and asked what f(*something else*) is, simply replace every instance of x in the "…" expression with whatever is now in the parentheses.
  - For example, let's say you have $f(x) = |x|^2 - 4$. If asked the value of $f(-2)$, then $f(-2) = |-2|^2 - 4 = 2^2 - 4 = 4 - 4 = 0$.
- Follow the same process for "strange operators"—symbols you don't know.
  - Say the example includes $x\Delta y$ and you're asked what $a\Delta 2x$ is: just replace "x" and "y" with "a" and "$2x$." So if $x\beta y = 3x + y^2$, then $5\beta 2 = 3(5) + (2)^2$.

Inequalities

- They can be treated like regular equations, with the following exception: multiplying or dividing an inequality by a negative number reverses the sign of the inequality.
  - $x < 1 \longrightarrow -x > 1$
- If $w < x < y$, then break it up into $w < x$ and $x < y$.
  - In that case, we also know that $w < y$.
- If $a < b$ and $c < d$, then $a + c < b + d$. However, this does not hold for subtracting, multiplying, or dividing.
- If $|x| < 3$, then $-3 < x < 3$; if $|x| > 3$, then $x > 3$ or $x < -3$.

ALGEBRA

Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

1. If $f(x) = x^3 - 5$ and $f(k) = 3$, then $k =$
   a. -22
   b. 2
   c. 4
   d. 6
   e. 22

2. Which of the following is a root of the equation $2x^2 - 20x = 48$?
   a. -4
   b. 2
   c. 6
   d. 8
   e. 12

3. If $2x - 3y = 6$, then $6y - 4x =$
   a. -12
   b. -6
   c. 6
   d. 12
   e. Cannot be determined

4. If $3x < 2y < 0$, which of the following must be the greatest?
   a. $2y - 3x$
   b. $3x - 2y$
   c. $-(3x - 2y)$
   d. $-(3x + 2y)$
   e. 0

ALGEBRA

Answers and Explanations

Click the play button for a video explanation.

1. Answer: b. 2

$f(x) = x^3 - 5$

$f(k) = k^3 - 5$

$3 = k^3 - 5$

$8 = k^3$

$2 = k$

2. Answer: e. 12

$2x^2 - 20x - 48 = 0$

$2(x^2 - 10x - 24) = 0$

$2(x - 12)(x + 2) = 0$

$x - 12 = 0 \longrightarrow x = 12$

$x + 2 = 0 \longrightarrow x = -2$

FAQ: Why doesn't $2(x-4)(x-6)$ work?

Great question! $(x-4)(x-6)$ results in $x^2 - 10x + 24$. Note that the last term is + 24, but we need - 24. Pay close attention to the positive and negative signs when factoring equations.

3. Answer: a. -12

$2x - 3y - 6$

$4x - 6y = 12$ If $a - b = c$ then $b - a = -c$

$-4x + 6y = -12$

$6y - 4x = -12$

4. Answer: d. $-(3x + 2y)$

$3x < 2y < 0 \longrightarrow 3x = -2,\ 2y = -1$

a. $2y - 3x = (-1) - (-2) = 1$

b. $3x - 2y = (-2) - (-1) = -1$

c. $-(3x - 2y) = -((-2) - (-1)) = 1$

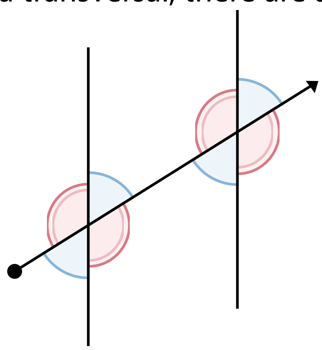
d. $-(3x + 2y) = -((-2) + (-1)) = \mathbf{3}$

e. $0 = 0$

GEOMETRY

Angles

- A right angle is made up of 90 degrees.
- A straight line is made up of 180 degrees.
- If two lines intersect, the sum of the resulting four angles equals 360 degrees.
- If two parallel lines are cut by a transversal, there are a total of two resulting angle measurements.



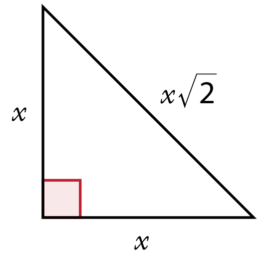
Polygons

A polygon is any figure with three or more sides (*e.g.*, triangles, squares, octagons, *etc.*).

- total degrees $= 180(n - 2)$, where n = # of sides

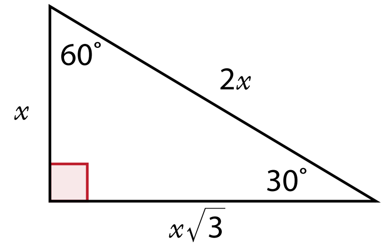
Triangles

- area $= \frac{1}{2} b \times h$
- Side a + Side $b >$ Side c
- Side a - Side $b <$ Side c
- Any given angle of a triangle corresponds to the length of the opposite side. The larger the degree measure of the angle, the larger the length of the opposite side.
- A **right triangle** has a right angle (a 90 degree angle); the side opposite the right angle is called the hypotenuse and is always the longest side.
- For a right triangle with legs a and b and hypotenuse c: $a^2 + b^2 = c^2$. This is called the Pythagorean Theorem.
- Certain right triangles have sides with all integer lengths. These sets of numbers are called Pythagorean triples, and you should memorize some of them: 3-4-5, 5-12-13, and 8-15-17. A multiple of a Pythagorean triple is also a Pythagorean triple (*e.g.*, 6-8-10).
- A 45˚-45˚-90˚ triangle has sides in a ratio of $x : x : x\sqrt{2}$, with $x\sqrt{2}$ as the hypotenuse.

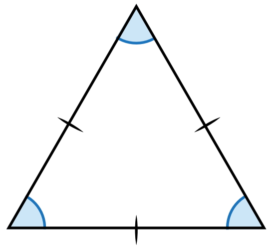


GEOMETRY

- This is also categorized as an isosceles triangle. A 30˚-60˚-90˚ triangle has sides in a ratio of $x : x\sqrt{3} : 2x$, with the $1x$ side opposite the 30 degree angle and $2x$ as the hypotenuse.



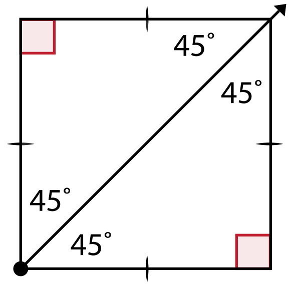
- An **equilateral triangle** has three equal sides. Each angle is 60 degrees.



Parallelograms

Squares:

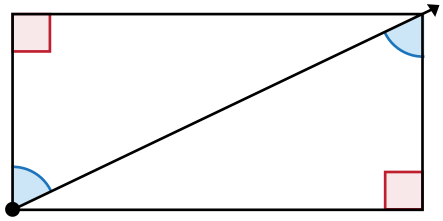
- Perimeter $= 4s$, where s = side
- Area $= s^2$
- A line drawn from one corner of a square to its opposite will create two 45-45-90 triangles.



- The diagonals of a square bisect each other, forming four 90 degree angles

Rectangles:

- Area $= l \times w$, where l = length and w = width
- Perimeter $= 2l + 2w$
- A line drawn from one corner of a rectangle to its opposite will create two congruent (equal) triangles.



GEOMETRY

Others:

- The diagonals of any rhombus bisect one another, forming four 90 degree angles.
- The area of any parallelogram can be found multiplying base x height (the base always forms a right angle with the height).

Trapezoids

- Area $= \frac{\text{Base}_1 + \text{Base}_2}{2} \times \text{height}$, where bases one and two are the two parallel sides.

3-D Shapes

Rectangular Solids (including cubes):

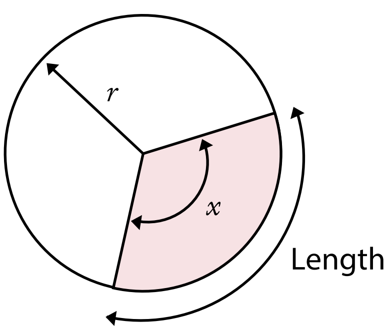
- Volume $= \text{height} \times \text{depth} \times \text{width}$
- Surface area $= 2(\text{height} \times \text{width}) + 2(\text{width} \times \text{depth}) + 2(\text{height} \times \text{depth}) =$ total areas of each rectangle
- The volume of a cube and the surface area of a cube are equal when s = 6.

Cylinders:

- Volume $= r^2 \pi h$

Circles

- Area $= \pi r^2$
- Circumference $= 2\pi r$
- A circle has 360 degrees.
- An **arc** is the portion of the circumference of a circle in x degrees of the circle.
  - Arc length $= \frac{x}{360} 2\pi r$



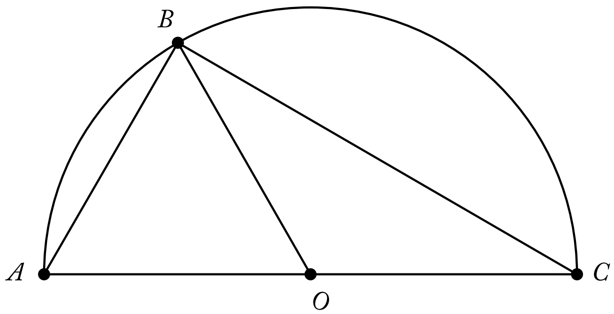
- A **sector** is a slice of a circle created by a central angle extending out to an arc. The shaded region in the circle above is a sector.
  Area of sector $= \frac{x}{360} \pi r^2$

GEOMETRY

Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

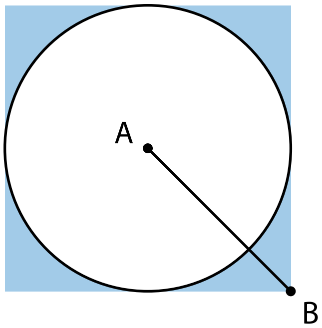
1. O is the center of the semicircle. If $\angle BCO = 30^\circ$ and $BC = 6\sqrt{3}$, what is the area of triangle ABO?



   a. $4\sqrt{3}$
   b. $6\sqrt{3}$
   c. $9\sqrt{3}$
   d. $12\sqrt{3}$
   e. $24\sqrt{3}$

2. If the ratio of the volume of cube A to the volume of cube B is 1 to 8, what is the ratio of the surface area of cube A to the surface area of cube B?
   a. $1:\sqrt{2}$
   b. 1:2
   c. $1:2\sqrt{2}$
   d. 1:4
   e. 1:8

3. A is the center of the circle, and the length of AB is $4\sqrt{2}$. The blue shaded region is a square. What is the area of the shaded region?



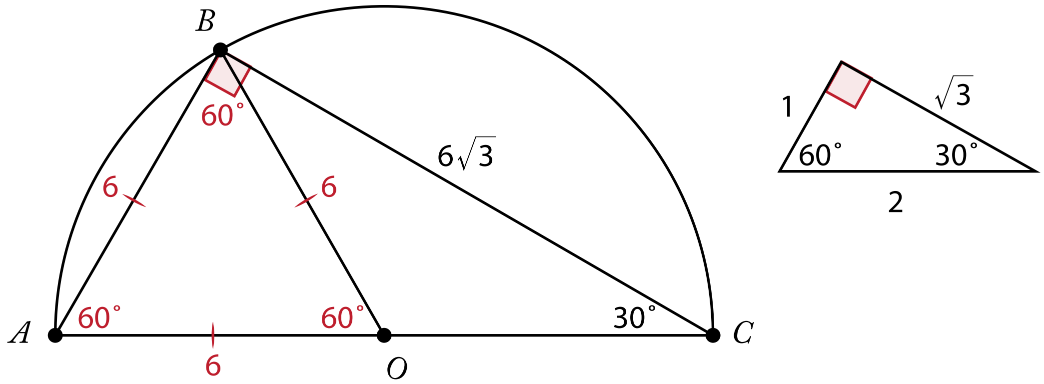
   a. $4(4 - \pi)$
   b. $4(8 - \pi)$
   c. $8(2 - \pi)$
   d. $8(8 - \pi)$
   e. $16(4 - \pi)$

GEOMETRY

Answers and Explanations

Click the play button for a video explanation.

1. Answer: c. $9\sqrt{3}$



$$\text{Area of equilateral } \Delta = \frac{\sqrt{3}}{4}(\text{side}^2)$$

$$\text{Area} = \frac{\sqrt{3}}{4}(6^2)$$

$$= \frac{\sqrt{3}}{4}(36)$$

$$= 9\sqrt{3}$$

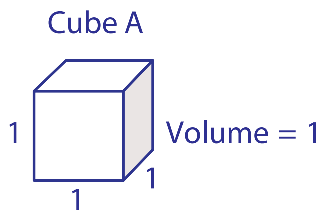
FAQ: How do we know that angle ABC is a right angle?

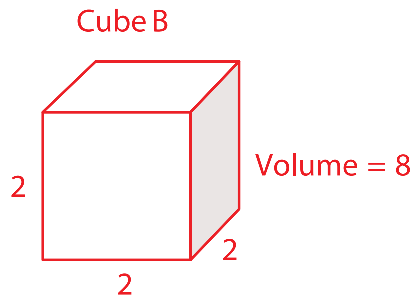
There are two things we need to know to understand the right angle:

- We are told O is the center, and any line that goes from the center of a circle to the circle edge is a radius.
- When you have an inscribed angle in a triangle that connects to the diameter, that inscribed angle is always 90 degrees.

GEOMETRY

2. Answer: d. 1:4



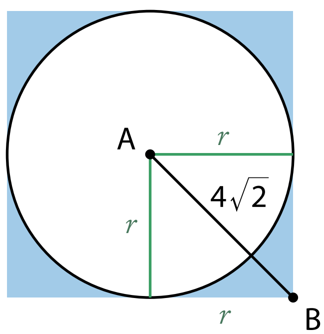


$$\begin{aligned}\text{Surface area} &= 6 \times (\text{area of one face})\\ &= 6 \times (1)\\ &= 6\end{aligned}$$

$$\begin{aligned}\text{Surface area} &= 6 \times (\text{area of one face})\\ &= 6 \times (4)\\ &= 24\end{aligned}$$

$$\begin{aligned}\text{Ratio of surface areas} &= 6:24\\ &= 1:4\end{aligned}$$

3. Answer: e. $16(4 - \pi)$



$$r^2 + r^2 = \left(4\sqrt{2}\right)^2$$
$$2r^2 = 32$$
$$r^2 = 16$$
$$r = 4 \longrightarrow \text{side} = 8$$

Shaded area	=	Square area	-	Circle area		
		side^2		πr^2		
		8^2		$x \times 4^2$		
		64	-	16π	$\longrightarrow$	$16(4 - \pi)$

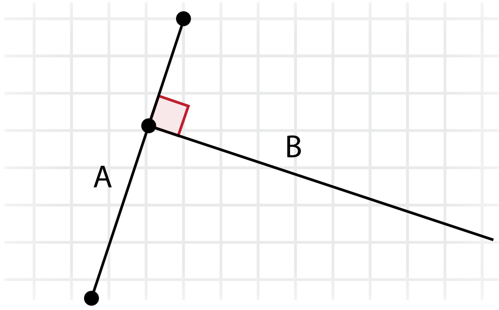
COORDINATE GEOMETRY

Lines

- Any line can be represented by $y = mx + b$, where m is the slope and b is the y-intercept. This is called **slope-intercept form**.
- The slope of a line is the difference in the y values of a pair of coordinates divided by the difference in the x values:

$$slope = m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

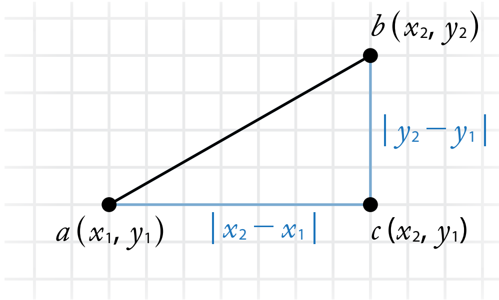
- To find the y-intercept, plug in zero for x and solve for y; to find the x-intercept, plug in zero for y and solve for x.
- An equation like $x = 3$ is a vertical line at $x = 3$; an equation like $y = 4$ is a horizontal line at $y = 4$.
- If given two points and asked to find the equation of a line that passes through them, first find the slope using the above formula, then plug one of the points into $y = mx + b$ and solve for b.
- The slope of a perpendicular line is the negative reciprocal of the first line.
    - For example, if the slope of line A is 3, and line B is perpendicular, then the slope of B is -⅓.



The Distance Formula

For finding the distance between (x_1, y_1) and (x_2, y_2)

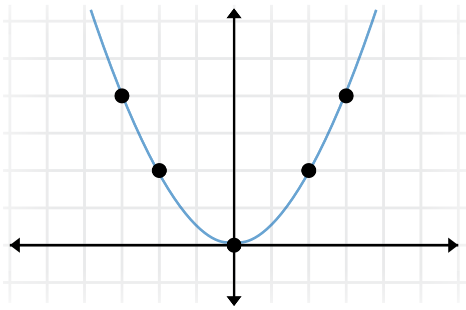
- $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Can be thought of as the hypotenuse of a triangle with legs parallel to the x- and y-axes. Rather than memorizing this formula, you can just use the Pythagorean theorem.
    - For example:



COORDINATE GEOMETRY

Quadratics

- This is the format of a quadratic equation: $y = ax^2 + bx + c$.
- The graph of a quadratic equation is a symmetrical shape called a **parabola**, which open upwards if $a > 0$ and down if $a < 0$.



Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

1. In the xy-coordinate system, line k has slope 1/2 and passes through point (0, 5). Which of the following points **cannot** lie on line k?
   a. (-10, 0)
   b. (8, 9)
   c. (3, 6.5)
   d. (-2, 2)
   e. (-8, 1)

2. In the xy-coordinate system, the distance between the point (0,0) and point P is . Which of the following could be the coordinates of point P?
   a. (4, 7)
   b. (4, 10)
   c. (5, 6)
   d. (6, 2)
   e. (20, 20)

COORDINATE GEOMETRY

Answers and Explanations

Click the play button for a video explanation.

1. Answer: d. (-2, 2)

$y = mx + b$ $m =$ slope, $b = y$ intercept

$y = \frac{1}{2}x + 5$

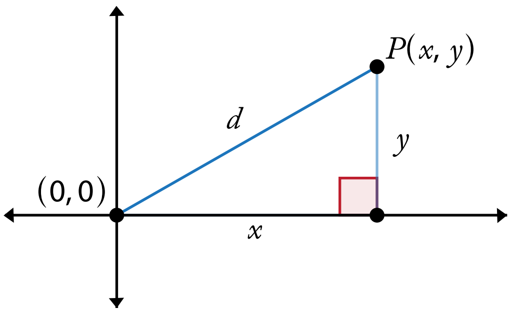
a. (- 10, 0) ⟶ $0 = \frac{1}{2}(-10) + 5$

b. (8, 9) ⟶ $9 = \frac{1}{2}(8) + 5$

⋮

d. (- 2, 2) ⟶ $2 = \frac{1}{2}(-2) + 5 = 4$ ✘

2. Answer: d. (6, 2)



$x^2 + y^2 = d^2$

$\sqrt{x^2 + y^2} = d$

$\sqrt{x^2 + y^2} = \sqrt{40} \longrightarrow x^2 + y^2 = 40$

a. $(4, 7) \longrightarrow 4^2 + 7^2 = 65$

⋮

d. $(6, 2) \longrightarrow 6^2 + 2^2 = 40$ ✔

WORD PROBLEMS

Translating Word Problems into Equations

Be especially careful to follow the logical order of the word problem when creating an equation. Below you will find words that are associated with a specific type of mathematical operation.

Addition

- the sum of
- the total of
- combined with
- increased by
- and

Subtraction

- the difference between
- less than / fewer than
- decreased by

Multiplication

- the product of
- ___ times as many
- of

Division

- per / a
- out of
- numerator = the top number
- denominator = the bottom number

Distance, Rate, and Time

- distance $=$ rate $\times$ time
- rate $= \frac{\text{distance}}{\text{time}}$
- time $= \frac{\text{distance}}{\text{rate}}$
- average speed $= \frac{\text{total distance traveled}}{\text{total time}}$
    - Average speed **is not** found by averaging two speeds.

Work Rate

- $\frac{1}{\text{TotalWork}} = \frac{1}{\text{WorkRate}_1} + \frac{1}{\text{WorkRate}_2}$
- output $=$ rate $\times$ time

Sequences

Arithmetic Sequences:

- Follow the pattern of $a, a+d, a+2d, a+3d, \ldots, a+(n-1)d$
    - For example: 3, 7, 11, 15, 19, 23, 27
    - The value of term n is $a+(n-1)d$

WORD PROBLEMS

- To find the sum of an arithmetic sequence, add the first and nth terms, multiply by n, then divide by two.
    - The sum of the example above is $\frac{([3+27]\times 7)}{2} = \frac{(30\times 7)}{2} = \frac{210}{2} = 105$

Geometric Sequences:

- Follow the pattern of $a, a\times d, a\times d^2, a\times d^3, \ldots, a\times d^{(n-1)}$
    - For example: 3, 6, 12, 24, 48, 96, 192
    - The value of term n is $a\times d^{(n-1)}$

Statistics

Average (mean):

For a set of n numbers: $\frac{\text{total sum}}{n}$

- *E.g.*, in the list 2, 2, 4, 4, 4, 5, 7, 7, 10, 10 the mean is $\frac{55}{10} = 5.5$

Median:

The middlemost value when numbers are arranged in ascending order; for an even amount of numbers, take the average of the middle two.

- *E.g.*, in the list 2, 2, 4, 4, 4, 5, 7, 7, 10, 10 the median is $\frac{(4+5)}{2} = 4.5$
- If the numbers in a set are evenly spaced, then the mean and median of the set are equal. In 30, 35, 40, 45, 50, the mean and median are both 40.

Mode:

The number that occurs most frequently.

- *E.g.*, in the list 2, 2, 4, 4, 4, 5, 7, 7, 10, 10 the mode is 4

Range:

Greatest value – least value

- *E.g.*, in the list 2, 2, 4, 4, 4, 5, 7, 7, 10, 10 the range is $10 - 2 = 8$

Weighted average:

$(\text{proportion})\times(\text{group A average}) + (\text{proportion})\times(\text{group B average}) + \ldots$

- *E.g.*, in a group of 2 girls and 4 boys, if the girls' average height is 60 inches and the boys' average height is 66 inches, then the total average is $\left(\frac{2}{6}\times 60\right) + \left(\frac{4}{6}\times 66\right) = 20 + 44 = 64$
- Weighted averages are very rare on the SAT, and the formula isn't completely necessary. Just be sure to find any new average using totals; NEVER simply average two averages.

WORD PROBLEMS

Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

1. When positive integer x is divided by 11, the quotient is y and the remainder is 4. When $2x$ is divided by 8, the quotient is $3y$ and the remainder is 2. What is the value of $13y - x$?
   a. -4
   b. -2
   c. 0
   d. 2
   e. 4

2.

Score	Number of students
40	1
55	2
70	3
x	4

Ten students took a test, and the distribution of scores is shown on the frequency table. If the average (arithmetic mean) score is 62, what is the value of x ?
   a. 62
   b. 65
   c. 71
   d. 76
   e. 83

WORD PROBLEMS

Answers and Explanations

Click the play button for a video explanation.

1. Answer: d. 2

$$2x \div 8 = 3y(2) \longrightarrow 24y + 2 = 2x \longrightarrow 24y = 2x = -2$$

$$x \div 11 = y(4) \longrightarrow 11y + 4 = x \longrightarrow -\ 11y - x = -4$$

$$13y - x = 2$$

2. Answer: b. 65

Score	Number of students
40	1
55	2
70	3
x	4

$$\text{Average} = \frac{\text{Sum of all 10 scores}}{10}$$

$$62 = \frac{40 + 55 + 55 + 70 + 70 + 70 + x + x + x + x}{10}$$

$$620 = 40 + 55 + 55 + 70 + 70 + 70 + x + x + x + x$$

$$620 = 360 + 4x$$

$$260 = 4x$$

$$65 = x$$

COUNTING

Fundamental Counting Principle

If a task is comprised of stages, where…

- One stage can be accomplished in a ways
- Another can be accomplished in b ways
- Another can be accomplished in c ways

…and so on, then the total number of ways to accomplish the task is $a \times b \times c \times \ldots$

Approach to a Counting Problem

1. Identify/list possible outcomes
2. Determine whether the task can be broken into stages
3. Determine the number of ways to accomplish each stage, beginning with the most restrictive stage(s)
4. Apply the Fundamental Counting Principle

Factorial Notation

$n! = n \times (n - 1) \times (n - 2) \times \ldots \times 3 \times 2 \times 1$

- n unique objects can be arranged in $n!$ ways. Example: There are 9 unique letters in the word "wonderful", so we can arrange its letters in $9 \times 8 \times 7 \ldots = 362{,}880$ ways.

Restrictions

- (Number of ways to follow a rule) = (number of ways ignoring the rule) - (number of ways to break the rule)

Combinations

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

- When the order does not matter—for example, picking any 3 friends from a group of 5.

Permutations

- $_nP_r = \frac{n!}{(n-r)!}$
- When the order does matter. For example, how many ways you could you order 3 letters from the word PARTY? The formula for permutations is very similar to combinations; we just remove the $r!$ from the denominator.
- ${}_5C_3 = \frac{5!}{(5-3)!} = 60$

COUNTING

Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

1. If k is the greatest positive integer such that 3^k is a divisor of 15! then $k =$
    a. 3
    b. 4
    c. 5
    d. 6
    e. 7

2. The probability is 0.6 that an "unfair" coin will turn up tails on any given toss. If the coin is tossed 3 times, what is the probability that **at least** one of the tosses will turn up tails?
    a. 0.064
    b. 0.36
    c. 0.64
    d. 0.784
    e. 0.936

COUNTING

Answers and Explanations

Click the play button for a video explanation.

1. Answer: d. 6

$$15! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15$$
$$= 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15$$
$$= 1 \times 2 \times 3 \times 4 \times 5 \times (3 \times 2) \times 7 \times 8 \times (3 \times 3) \times 10 \times 11 \times (3 \times 4) \times 13 \times 14 \times (3 \times 5)$$
$$= 3^6(1 \times 2 \times 4 \times 5 \times 2 \times 7 \times 8 \times 10 \times 11 \times 4 \times 13 \times 14 \times 5)$$

2. Answer: e. 0.936

$$P(\text{tails}) = 0.6 \longrightarrow P(\text{heads}) = 1 - 0.6 = 0.4$$

$$\begin{aligned} P(\text{at least 1 tails}) &= 1 - P(\text{no tails}) \\ &= 1 - P(\text{all heads}) \\ &= 1 - P(H_1 \text{ and } H_2 \text{ and } H_3) \\ &= 1 - [P(H_1) \times P(H_2) \times P(H_3)] \\ &= 1 - [0.4 \times 0.4 \times 0.4] \\ &= 1 - [.064] \\ &= 0.936 \end{aligned}$$

FAQ: Isn't the probability of at least one tails (0.6)(0.4)(0.4) + (0.6)(0.6)(0.4) + (0.6)(0.6)(0.6)? What am I missing?

A: The basic thinking is correct, but there are actually more than 3 cases here which would satisfy our requirement for at least one tails. (H = heads, T = tails):

One tails: THH, HTH, HHT

Two tails: TTH, THT, HTT

Three tails: TTT

Now we need to analyze the probability for each of these cases and add them:

THH = (0.6 × 0.4 × 0.4) = .096

HHT = (0.4 × 0.4 × 0.6) = .096

THT = (0.6 × 0.4 × 0.6) = .144

TTT = (0.6 × 0.6 × 0.6) = .216

HTH = (0.4 × 0.6 × 0.4) = .096

TTH = (0.6 × 0.6 × 0.4) = .144

HTT = (0.4 × 0.6 × 0.6) = .144

When you add all of those up, you get 0.936 as your answer. Of course, this is much more complicated than noting that there's only one possible scenario in which NO tails are flipped (HHH), finding that probability and then subtracting it from 1.

PROBABILITY

Probability of Event A

- $P(A) = \dfrac{\text{number of outcomes where } A \text{ occurs}}{\text{total number of outcomes}}$

Events A & B (if Independent Events)

- $P(A \text{ and } B) = P(A) \times P(B)$
  - For example, if given a deck of cards, the probability of drawing a heart, returning that card to the deck, then drawing a heart again.

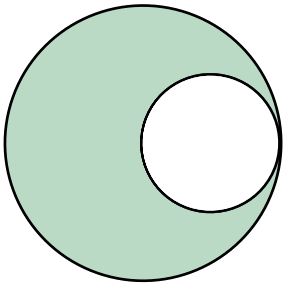
Events A & B (if Dependent Events)

- $P(A \text{ and } B) = P(A) \times P(B \mid A)$
  - $P(B \mid A)$ is the probability that B occurs given that A occurs.
  - If given a deck of cards, the probability of drawing a heart, keeping that card outside the deck, then drawing a heart again.

Probability of Shaded Regions

- The probability of a point lying within a specific region of a figure is the fraction of the total area which that region takes up.

  For example,



$$\frac{\text{area of shaded region}}{\text{area of entire larger circle}} = \text{probability of shaded region}$$

PROBABILITY

Practice Questions

For answers and explanations, click a multiple choice option or scroll to the next page.

1. Each circle has center O. The radius of the smaller circle is 2 and the radius of the larger circle is 6. If a point is selected at random from the larger circular region, what is the probability that the point will lie in the shaded region?

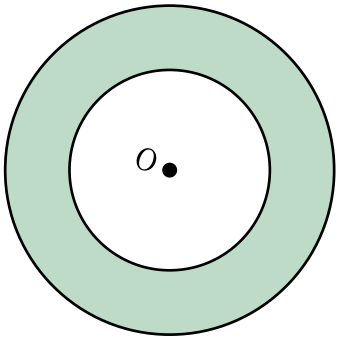
   a. $\frac{1}{9}$

   b. $\frac{1}{6}$

   c. $\frac{2}{3}$

   d. $\frac{5}{6}$

   e. $\frac{8}{9}$



PROBABILITY

Answers and Explanations

Click the play button for a video explanation.

1. Answer: e. $\frac{8}{9}$

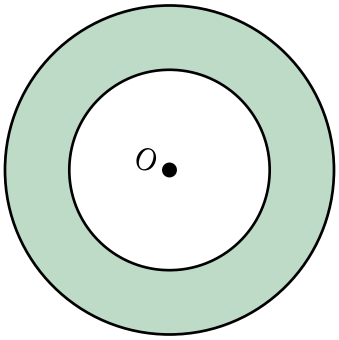
$$\text{Area} = \pi r^2$$

$$\text{Small Circle: Area} = \pi(2)^2 = 4\pi$$

$$\text{Large Circle: Area} = \pi(6)^2 = 36\pi$$

$$\text{Unshaded portion} = \frac{4\pi}{36\pi} = \frac{1}{9}$$

$$\text{Shaded portion} = \frac{8}{9}$$



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